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Private Observation, Tacit Collusion and Collusion with Communication

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Private Observation, Tacit Collusion and Collusion with Communication*

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Abstract

The paper studies the role of communication in facilitating collusion. The situation of infinitely repeated Cournot competition in the presence of antitrust enforcement is considered. Firms observe only their own production levels and a common market price. The price is assumed to have a stochastic component, so that a low price may signal either deviations from collusive output levels or a 'downward' demand shock. The firms choose between tacit collusion and collusion with communication. Communication implies that the firms meet and exchange information about past outputs and is assumed to be the only legal proof of cartel behavior. The antitrust enforcement takes the form of an exogenous probability to detect the meetings, in which case the firms are sued for cartel behavior and pay a fine. Tacit collusion is assumed to provide no grounds for the legal action but involves inefficiencies due to the lack of complete information about individual output levels. It is shown that there exists a range of discount factors where collusion with communication constitutes the most profitable collusive strategy.

Keywords: Collusion, Communication, Private Information.

JEL Classification: D82, L41.

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1 Introduction

The paper explores the role of communication in facilitating collusion in an infinite-horizon setting, where firms' individual actions are private information and each firm can make only statistical inference about the behavior of its rivals. Communication is assumed to allow firms to share private information about their individual actions and thus helps sustain collusion. This type of communication, however, may not be always beneficial because it often generates an incontestable proof of collusive behavior which can be revealed by competition authorities (CA). In the situation where firms can choose between tacit collusion and collusion with communication, the paper shows that the most profitable collusive strategy may involve communication for a range of intermediate values of the discount factor.

The conventional economic analysis of collusive behavior implicitly assumes that firms form a tacit agreement about each other's actions[1] based on information which is commonly observed and readily available to every party. Firms cannot sign any legal contract[2] aimed to sustain collusion because it is per se prohibited, yet they may enforce a cartel agreement in the context of an infinitely repeated competition game. Tacit collusion thus implies that there is no need for collusive firms to communicate and, therefore, requires no explicit accounting of their actions.

In practice, however, there is a strong evidence that collusive firms do communicate.[3] Despite the fact that communication does not allow them to legally sign a collusive contract, so that the sustainability of collusion is still an issue, it has been found that they *explicitly* collude by holding regular meetings and keeping records of each other's actions. Thus, the role of communication in collusion needs further clarification.

In their excellent study of the Sugar Institute Case, Genesove and Mullin (2001) highlight the deficiencies in the classical theory of collusion and stress the reasoning behind firms' regular meetings. They find that communication mainly serves two purposes. First, it *enhances the efficiency* of a cartel outcome and, second what is more important, it *strengthens the sustainability* of the cartel. In the latter case, communication facilitates detection of cartel deviators and eliminates mistakes in establishing the fact of cheating.

The recent paper by Athey and Bagwell (2001) on price collusion with private information about unit costs touches upon the issue of efficiency. They find that the optimal collusion scheme involves a sophisticated mechanism design aimed to elicit the true type of each firm and, thus, implies

[1] In other words, the firms agree once and forever on a particular equilibrium strategy from the set of all possible equilibrium strategies of the supergame.

[2] For example, an agreement to set the monopoly price or collectively produce the monopoly output every period would be legally prohibited.

[3] See, for example, the Commision decision on the choline chloride cartel (Case COMP/E-2/37.533) or the citric acid cartel (Case COMP/E-1/36.604).

communication. In particular, the paper shows that productive and pricing efficiency necessarily requires consideration of asymmetric schemes:[4] in order to induce a high-cost firm to reveal its type truthfully and thus abstain from making any sales "today" it should be promised a reward "tomorrow". This in turn implies that this firm should receive a higher market share tomorrow provided that both firms are equally efficient.

A potential advantage of communication is explored in Compte (1998) and Kandori and Matsushima (1998). Both papers develop basically the same model in which the actions of players are private information and each player observes *private* and imperfect signals of its rivals' past play. The latter fact causes a serious difficulty for the sustainability of collusion: since the players receive *diverse* information about past history of the play they may end up having different expectations about what might have happened. Communication in such a setup allows the players to exchange information of the private signals and thus aims to enhance the sustainability of collusion. However, as the authors acknowledge, they are not able to characterize the set of equilibria when there is no communication, and therefore to evaluate the potential benefits which the players derive from communication in collusion.

The focus of the present paper is on the role of communication in facilitating collusion. Following the approach of Green and Porter (1984), I develop a model where firms' individual outputs constitute private information. Firms receive an imperfect signal about each other's behavior through the realization of the stochastic market price which is *publicly* observable. The main obstacle for collusion comes from the fact that a low price may signal either deviations from collusive output levels or a 'downward' demand shock. I modify the Green and Porter approach by allowing firms to hold meetings and exchange private information about each firm' past behavior before they produce their outputs. Communication thus helps resolve confusion about the past play. Despite the benefits of communication, yet it is not completely innocuous: cartel participants are under the constant threat of legal prosecution because their meetings can be detected by competition authorities, in which case they are sued and pay a fine.

There are many cartel cases in which firms have been found to exchange private information of their past outputs during cartel meetings. In the Choline Chloride Case, for example, the Decision of the European Commission states (paragraph 69):

> Whether the agreed actions were being accomplished in prac-

[4] However, in their paper Athey et al. (1997) develope a model with continuum of cost types and study optimal symmetric schemes. Their main result is that if the cost distribution function is log-concave then the parties may find it more profitable to sacrifice efficiency benefits by adopting the rigid-pricing scheme, i.e. in each period the firms select the same price whatever their cost levels, and thus abandon *per se* communication whatsoever.

> tice was regular checked. The parties agreed to meet every six months to monitor, discuss and correct any problems. In these follow-up meetings, the parties compared information on sales actually made during the last period and discussed whether the group's goals were being achieved.[5]

As Kuhn (2001) notices, in the Green and Porter model the observation of (dis)aggregate output(s) suffices to enhance the sustainability of collusion. The present paper provides a formal analysis of the benefits which firms can derive from the exchange of private information in the presence of the antitrust enforcement.

Contrary to Athey and Bagwell (2001), where firms share information about the *current* period in order to increase the profitability of collusion, in the present model the purpose of communication is to establish compliance with the collusive agreement and thus strengthen the sustainability of collusion. Another difference is the nature of information transmitted during a meeting. In their model individual cost types constitute a piece of soft information and, therefore, its reference to the past has little, if any, use for the collusive parties. Moreover, since such information has no direct impact on firms' current and future objectives, little can be done to elicit it truthfully.

Similarly to Compte (1998) and Kandori and Matsushima (1998), in the present model firms can communicate information about the past behavior. The difference, however, is that in their model firms exchange soft information, while here they exchange hard information. Furthermore, in contrast to their approach, it is possible to derive the best collusive strategy when communication is absent and thus evaluate the potential benefits of communication for collusive firms.

The main assumption of the model is that firms have hard evidence about private actions. There is a rationale behind the fact that a firm may not be able to distort or forge its report about its past behavior because, say, verification of the report may be costless. For example, in the Choline Chloride Case, it has been established that collusive firms verified the commercially sensitive information through the European Trade Association for the Chemical Sector (CEFIC). In other instances, it has been found that collusive firms may resort to establishing interior auditing schemes at the most senior levels of management to monitor individual volumes of sales, as in the Vitamin Case,[6] or they may ask an independent auditing company to perform a similar task. Taking this into account, the main question the parties confront is whether to comply with the collusive agreement by submitting a report about their past behavior.

[5] See the Commision Decision on Case COMP/E-2/37.533.

[6] See the Commision Decision on Case COMP/E-1/37.512.

This type of communication, however, may leave incriminating evidence of cartel behavior, which can be discovered by the CA.[7] If the meeting is detected, firms are sued and pay a fine. In contrast, if they choose tacit collusion then there is eventually no way to incriminate them for cartel behavior. Indeed, the main proof of firms' engagement in cartels often comes from revealed notes, faxes, e-mails and other records of meetings. Moreover, there is a strong belief among CA officials that if firms collude tacitly, they can never be summoned to the court because of the lack of hard incriminating evidence. As the case-law shows, such as in the EU Wood Pulp Case, pure economic reasoning has been often found not sufficient from a jurisdictional point of view to accuse undertakings in collusive behavior.

Although tacit collusion has the advantage of involving no legal action from the CA, it is not completely innocuous for collusive firms. Since individual actions are only privately observable, they have to incur informational costs in designing a collusive scheme. In particular, there is more scope for deviations, which, in turn, makes tacit collusion more fragile.

First, firms may deviate *openly,* that is they may optimally respond to the collusive output and, thereby, clearly reveal cheating. This type of deviation is common to both tacit and explicit collusive schemes. Second, firms may cheat in a *hidden* way, that is they may opt for a suboptimal response to the collusive output in order to induce the likelihood of being detected.

The analysis is based on the comparison of the best payoffs obtained, correspondingly, in the tacit and explicit collusion schemes. It is found that in choosing the most profitable type of collusion, firms face the following tradeoff. Without communication collusion always involves informational costs due to imperfect knowledge of firms' individual actions while with communication collusive profits are reduced because of the fine imposed in the event the meeting is detected. The central result of the paper is that as long as the punishment for cartel behavior is not too large, there exists an intermediate range of discount factors where collusion with communication constitutes the best collusive strategy.

The intuition is as follows. In explicit collusion firms save on informational costs but always bear the risk of being fined for illegal behavior. In contrast, in tacit collusion firms are never exposed to legal punishment but instead incur informational costs which can vary with the value of the discount factor. In particular, when the discount factor is large, information costs are absent because firms can deter all deviations and sustain perfect collusion. When the discount factor is small, there are no information costs either because in this case a cheating firm would prefer the open deviation.

[7] For example, in the Carbonless Paper Case, cartel meetings were convened under the cover of the official meetings of the trade association and during the course of investigation the Commission received copies of the minutes of these meetings.

It is thus only for intermediate values of the discount factor that informational costs may impede collusion and firms may face the tradeoff in choosing between the tacit and explicit collusion schemes.

The rest of the paper is organized as follows. Section 2 develops the model and derives the optimal collusive scheme. Conclusions and some policy implications are presented in Section 3.

2 The Model

Two risk-neutral firms with identical unit costs of production c produce a homogeneous product and repeatedly compete *à la* Cournot.[8] Following the Green and Porter approach, it is assumed that the firms can observe only their own production levels. They face a common market price which is inversely related to the total industry output $Q_t = q_{1t} + q_{2t}$. The price is assumed to include some stochastic component and in order to simplify the exposition I confine to the linear price schedule with the intercept being a random variable

$$\widetilde{p}_t = a + \widetilde{\theta}_t - Q_t, \tag{1}$$

where the shocks $\{\widetilde{\theta}_t\}_{t=0}^{\infty}$ are identically and independently distributed across the time. It is assumed that $\widetilde{\theta}_t$ takes on two values, $\widetilde{\theta}_t \in \Theta = \{-\sigma, \sigma\}$ with equal probabilities and its realization is not directly observed by the firms. Let $P(Q) = \{p_L(Q), p_H(Q)\}$ denote the set of feasible price realizations given the total industry output Q, where $p_L(Q) = a - \sigma - Q$ and $p_H(Q) = a + \sigma - Q$. The firms are assumed to perfectly observe the realized market price $\widetilde{p}_t \in P(q_{1t} + q_{2t})$. Since a firm observes neither the output of its rival nor the true value of $\widetilde{\theta}$, it may face a nontrivial inference dilemma: for some realizations of $\widetilde{p}$ [9] it cannot infer with probability one what level of output the other firm has supplied on the market. The fact that it is impossible to make precise inference about privately taken actions constitutes the core problem for the sustainability of collusion. As it will be shown later, the firms have to sacrifice efficiency benefits in order to overcome this informational gap.

The CA is assumed to have no information regarding the relevant economic data of firms' costs, consumer demand or market shares. Therefore it cannot make any inference about market behavior from the mere price observation. The only proof of cartel behavior is assumed to come from the

[8] A simillar analysis would hold in case of Bertrand competion with differentiated products, where demand realizations for each product are determined by a common stochastic shock.

[9] In particular, it will be shown for the low-demand state realization p_L.

detection of firms' meetings.[10] In this setup, tacit collusion thus implies that cartel participants can never be sued and fined because of the lack of hard incriminating evidence. In contrast, if the firms explicitly collude and the meeting is discovered then the revealed evidence of exchange of information about past outputs provides an incontestable proof of the illegal behavior.

The single period expected profit of firm i is defined as

$$\pi_{it}^{\exp}(q_{it}, q_{jt}) = (p^{\exp}(q_{it} + q_{jt}) - c)\, q_{it} - K,$$

where $j \neq i$, K denotes fixed costs of production and $A \equiv a - c > 0$. It is easy to verify that the Nash equilibrium implies

$$q_i = q^{Nash} = \frac{1}{3}A \quad \text{and} \quad \pi_i^{\exp} \equiv \pi^{Nash} = \frac{1}{9}A^2 - K \text{ for } i = 1, 2.$$

To define the stage game G the following notation is employed. $\Xi = \{C, N\}$ is the set of decision choices common to both firms, where C and N imply communication and no communication, respectively. $P = [0, p_{\max}]$ is the set of all feasible price realizations and $S = [0, q_{\max}]$ is the set of output levels common to both firms.

The timing of the game G is thus as follows.

Stage 1. Each firm $i = 1, 2$ takes a decision $\zeta_i \in \Xi$ about whether to hold a meeting. After the decisions having been made, they are assumed to be known to every party. The meeting takes place if and only if both firms agree to communicate, i. e., $\zeta_1 = \zeta_2 = C$ and implies that the firms disclose their private output levels produced in the past.[11] In the event of no meeting, no information about past outputs is available.

Stage 2. Each firm $i = 1, 2$ chooses its output level $q_i \in S$, the shock $\widetilde{\theta}$ is realized and the market price $\widetilde{p} \in P$ is publicly observed.

Stage 3. The CA audits the industry. If communication has taken place, it finds the incriminated evidence with probability ρ in which case each firm is fined by the amount F, otherwise it finds nothing and no fine is imposed.

Stage 4. The payoffs of firms are realized.

The firms are supposed to play an infinitely repeated game $G^{\infty}(\delta)$ defined by the component game G and the discount factor $\delta \in (0, 1)$. A strategy Σ_i of firm i specifies for each period of time $t \geq 0$

(i) a decision variable $\zeta_{it} \in \Xi$ about the meeting at period t as a function of both firms' past decisions, the sequences of past prices and the firm's own past quantities:

[10] Here, the model abstracts from any reason to communicate other than to facilitate collusion.

[11] Notice that in this setup the decision of whether to meet is equivalent to the decision of whether to disclose private information because, by assumption, the firms cannot distort their reports.

$$\zeta_{it} : \Xi^{2(t-1)} \times P^{t-1} \times S^{t-1} \to \Xi,$$

for $t \geq 1$ and ζ_{i0} is given,

(ii) an output $q_{it} \in S$ as a function of both firms' past and current decisions about communication, the sequences of past prices and the firm's own past quantities:

$$q_{it} : \Xi^{2t} \times P^{t-1} \times S^{t-1} \to S,$$

for $t \geq 1$ and q_{i0} is given.

The expected one-period payoff of firm i is thus defined as follows:

$$v_{it}(\zeta_{it}, q_{it}; \zeta_{jt}, q_{jt}) = \begin{cases} \pi_{it}^{\exp}(q_{it}, q_{jt}) - \rho F, & \text{if} \quad \zeta_{it} = \zeta_{jt} = C, \\ \pi_{it}^{\exp}(q_{it}, q_{jt}), & \text{otherwise.} \end{cases}$$

Denote $\Sigma_{it} = (\zeta_{it}, q_{it})$ then a strategy Σ_i of firm i is $\Sigma_i = (\Sigma_{i1}, \Sigma_{i2}, ...)$. Each firm $i = 1, 2$ seeks to maximize the expected value of the discounted sum of its one-period payoffs

$$V_i = (1 - \delta) \sum_{t=0}^{\infty} \delta^t v_{it}(\Sigma_{it}, \Sigma_{jt}).$$

To simplify the analysis, only fully symmetric equilibria are considered. This implies $\Sigma_{1t} = \Sigma_{2t}$ for all $t \geq 0$ and thus $\pi_1^{\exp}(q, q) = \pi_2^{\exp}(q, q) \equiv \Pi^{\exp}(q)$.

In what follows I will derive the best (symmetric) payoffs respectively for tacit and explicit collusion. Finally, by comparing these payoffs, the optimal collusive scheme is obtained.

2.1 Tacit collusion

Tacit collusion implies no meetings, i.e., $\zeta_{1t} = \zeta_{2t} = N$ for any $t \geq 0$, and the objective of the firms is to find the most profitable collusive strategy in this case. The following assumption simplifies the analysis and, in particular, makes it easy to define the maximal punishment.

Assumption 1. $K = \frac{1}{9}A^2$ or $\pi^{Nash} = 0$.

It states that the minmax payoff (which is 0 here) is sustained simply by a reversal to the static Nash equilibrium.[12]

As it is proven in the Appendix, given that $\widetilde{\theta}$ is uniformly distributed, the best collusive strategy $\overline{\Sigma}^{nc}$ is stationary and consists in sticking to a collusive

[12] Under this interpretation, $\Pi^{\exp}(q) = (p^{\exp}(2q) - c)\, q - K$ constitutes the difference between the profit and the static Nash equilibrium profit.

output, q^{nc}, as long as the realized price, $\widetilde{p}$, is consistent with the target, $P(2q^{nc}) = \{p_L(2q^{nc}), p_H(2q^{nc})\}$. In case of any detected deviation the firms revert to the static Nash equilibrium forever. Formally, let Σ^{Nash} denote the strategy when every period the firms play the static Nash equilibrium then $\overline{\Sigma}^{nc}$ is defined as follows.

(i) At $t = 0$ agree on some output q^{nc}.

(ii) For any $t \geq 1$ produce q^{nc} if in the previous period the realized price $\widetilde{p} \in P(2q^{nc})$, otherwise play Σ^{Nash}.

Note that the best collusive strategy differs from the one obtained in Green and Porter (1984) and Abreu, Pearce and Stachetti (1986, 1990). Following their approach one might think that $\overline{\Sigma}^{nc}$ should specify a price war (at least for some periods) whenever the realized price is low, i.e., $\widetilde{p} = p_L(2q^{nc})$. Recall, however, that in their setting the support of possible price realizations is independent on the output levels which implies that firms' deviations are *never* detected. When the market price falls below some "trigger price" the firms switch on the price war in order to prevent any potential deviation. The principal difference from their setting is that the support $P(Q)$ of the realized market prices now *depends* on the output levels and is itself determined in equilibrium. In the model under study, for some output levels the firms can infer with probability one that cheating has occurred and therefore deviations are *partially* detected. As a result, the analogous strategy is not necessarily optimal.

Let $\overline{V}^c$ denote the payoff associated with the strategy $\overline{\Sigma}^{nc}$. Since $\overline{\Sigma}^{nc}$ is stationary then $\overline{V}^{nc} = \Pi^{\exp}(q^{nc})$. As the Appendix shows, in order that $\overline{\Sigma}^{nc}$ be an equilibrium strategy two types of no-deviation constraints must be satisfied.

First, a firm may deviate *openly* and thus clearly reveal cheating. In which case in the most profitable deviation it best responds to the collusive output q^{nc}. Since any deviation is most effectively deterred when the firms resort to the worst available punishment, the no-open deviation constraint takes the following form:

$$(1-\delta)\left[\underset{z}{Max}\ \pi^{\exp}(z, q^{nc}) - \Pi^{\exp}(q^{nc})\right] \leq \delta \Pi^{\exp}(q^{nc}). \qquad (\mathrm{IC}_{opn})$$

The left hand side of IC_{opn} is a one-period gain from deviation while the right hand side is the value of discounted losses from abandoning collusion forever afterwards.

Second, a firm may cheat in a *hidden* way, that is it may produce the output level that reduce the likelihood (down to $\frac{1}{2}$, here) of detection of cheating. As is well known for Cournot competition models, a cheating firm tends to expand its output production. Thus, the intuition suggests (and it is proven in the Appendix) that the most profitable deviation should involve a suboptimal increase of output, $r_H(q^{nc})$, so that when the demand is high the price level mimics the one of the low-demand state, i.e.,

$$r_H(q^{nc}): \; p_H\left(r_H(q^{nc}) + q^{nc}\right) = \; p_L\left(2q^{nc}\right).$$

Note that even when the firm produces $r_H(q^{nc})$, its deviation is revealed if the low-demand state is realized. Since in the latter case the firms switch on the punishment phase, the no-hidden deviation constraint takes the form:

$$(1-\delta)\left[\pi^{\exp}(r_H(q^{nc}), q^{nc}) - \Pi^{\exp}(q^{nc})\right] \leq \delta \frac{1}{2} \Pi^{\exp}(q^{nc}). \qquad (\text{IC}_{hdn})$$

In choosing between the open and hidden deviations a firm thus faces the following tradeoff. The gain from the hidden deviation is always lower because a cheating firm suboptimally responds to the collusive output. However, the discounted value of the future losses from abandoning collusion is also lower because its cheating is detected with probability $\frac{1}{2}$ only.

The analysis so far implies that in tacit collusion the firms choose q^{nc} that solves the following program:

$$\begin{array}{c} \overline{V}^{nc}(\delta) = \underset{q^{nc}}{Max} \Pi^{\exp}(q^{nc}) \\ s.t. \; \text{IC}_{opn} \text{ and } \text{IC}_{hdn} \text{ hold.} \end{array} \qquad (\text{P1})$$

Before proceeding with the characterization of a solution to P1, note that the optimal response $z^*(q^{nc})$ to the collusive output q^{nc} is

$$z^*(q^{nc}) = \arg\max_z \pi^{\exp}(z, q^{nc}) = q^{nc} + \frac{3}{2}(q^{Nash} - q^{nc}),$$

while the suboptimal response is $r_H(q^{nc}) = q^{nc} + 2\sigma$. In both cases, a cheating firm finds it profitable to increase the output. An important point, however, is that in the case of the hidden deviation the increase is determined by the variance σ of the shock $\widetilde{\theta}$. Thus, if σ is large enough then the suboptimal response can be very inefficient, i.e., $r_H(q^{nc})$ can be far beyond $z^*(q^{nc})$. In which case the hidden deviation is no longer attractive and the corresponding no-deviation constraint is per se irrelevant in program P1. To rule out such situation, I make

Assumption 2. $\sigma < \frac{1}{32\left(1-\frac{1}{\sqrt{2}}\right)} A.$

The following proposition establishes a key property of the solution to P1.

PROPOSITION 1. *There exist* $\widehat{\delta}_1$ *and* $\widehat{\delta}_2$ *such that* $0 < \widehat{\delta}_1 < \widehat{\delta}_2 < 1$ *and* IC_{hdn} *is binding in program P1 for any* $\delta \in (\widehat{\delta}_1, \widehat{\delta}_2)$.

Proof: see the Appendix.

The proposition thus states that the no-hidden deviation constraint is the relevant one for intermediate values of the discount factor. Intuitively, the open deviation yields a large one-period gain but provides no possibility to cheat in the future. In contrast, the hidden deviation yields a lower one-period gain but induces the probability to repeatedly cheat in the future. When the discount factor is small, a cheating firm values more the current period profit and thus finds the open deviation more profitable. On the other hand, when the discount factor is large, the firms can deter all deviations and sustain the monopoly outcome. Hence, it is only for intermediate values of the discount factor that the hidden deviation may weaken the sustainability of tacit collusion.

The exact analytical expressions of $\widehat{\delta}_1$ and $\widehat{\delta}_2$ as functions of σ are found to be complicated. To capture the impact of σ on $\widehat{\delta}_1$ and $\widehat{\delta}_2$ simulations were performed. Two representative examples when $A = 10$, $\sigma = 0,1$ and $\sigma = 0,4$ are shown on figures 1 and 2.

[Insert Figures 1 and 2 from the Appendix.]

The results confirm the intuition. For low values of σ the gain from the hidden deviation is small which implies that the hidden deviation is more attractive only when q^{nc} is close to q^{Nash}. This results to the narrow interval $(\widehat{\delta}_1, \widehat{\delta}_2)$ located in a range of small δ's. As σ increases, the interval $(\widehat{\delta}_1, \widehat{\delta}_2)$ broadens and shifts to the right. Finally, as σ approaches its upper bound, the interval $(\widehat{\delta}_1, \widehat{\delta}_2)$ shrinks and gradually disappears.

In order to gain the intuition about the magnitude of the losses due to imperfect information, consider the task of the collusive firms when the individual outputs are observable. Clearly, in this case only open deviations must be deterred and therefore the firms seek to solve the following program:

$$\begin{aligned} \overline{V}(\delta) = & \underset{q^{nc}}{Max} \Pi^{\exp}(q^{nc}) \\ & s.t. \text{IC}_{opn} \text{ holds.} \end{aligned} \tag{P2}$$

Since the analytical expressions of $\overline{V}^{nc}$ and $\overline{V}$ are complicated, I plot the two-dimensional graph of $1 - \frac{\overline{V}^{nc}}{\overline{V}}$ as a function of the discount factor δ and the ratio $\frac{\sigma}{A}$.

[Insert Figure 3 from the Appendix.]

As figure 3 shows, for some range of parameters the firms can lose more than 40% of their average intertemporal profit due to the lack of perfect information, i.e., when the no-hidden deviation constraint binds. In such a case they have strong incentives to eliminate the informational gap by means of communication.

2.2 Collusion with communication

Communication implies that the firms meet and exchange hard information about past outputs. Since communication is costly, the firms should use it in the most efficient way.

In particular, there is no use for a meeting at $t = 0$, since there is nothing to reveal yet. Similarly, for $t \geq 1$ for some price realizations the firms can detect deviations without communication. Indeed, if the firms agree to produce q^c at period t and the realized market price is not consistent with the targeted price, i.e., $\widetilde{p} \notin P(2q^c)$, then it is clear even *without* communication that cheating has occurred. Conversely, if the price level assigned to the high demand state is realized, i.e., $\widetilde{p} = p_H(2q^c)$, then one can infer *without* communication that *no* deviation has occurred either, since the hidden deviation that mimics the price assigned to the high demand state is never profitable.[13] Thus it is only when $\widetilde{p} = p_L(2q^c)$ that the firms are uncertain about each other's actions and may thus wish to communicate. The most profitable collusive strategy $\overline{\Sigma}^c$ with selective communication is formally defined as follows.

(i) At $t = 0$ do not meet, i.e. $\zeta_{i0} = N$, and produce some output q^c.

(ii) For any $t \geq 1$:

• if in period $t-1$ the realized price is $\widetilde{p}_{t-1} = p_H(2q^c)$, then in period t do not meet, $\zeta_{it} = N$ and produce q^c,

• if in period $t-1$ the realized price is $\widetilde{p}_{t-1} = p_L(2q^c)$, then in period t attend a meeting, $\zeta_{it} = C$,

• produce q^c in period t if the meeting has taken place and disclosed information is consistent with the collusive agreement, i.e., $q_{jt-1} = q^c$,

• play Σ^{Nash} if the meeting has not taken place when it should or the realized price is not consistent with the target, i.e., $\widetilde{p} \notin P(2q^c)$ or the meeting reveals that one firm has actually deviated.

Let V_H and V_L denote the expected present value of firm's profits given that the *previous* period price realizations are p_H and p_L respectively. Since the shock is uniformly distributed, one obtains

$$V_H = (1-\delta)\Pi^{\text{exp}} + \delta\left(\frac{1}{2}V_L + \frac{1}{2}V_H\right),$$

$$V_L = (1-\delta)(\Pi^{\text{exp}} - \rho F) + \delta\left(\frac{1}{2}V_L + \frac{1}{2}V_H\right).$$

Let $\overline{V}^c$ denote the payoff associated with the strategy $\overline{\Sigma}^c$. Since $\overline{\Sigma}^c$ prescribes no meeting at $t = 0$ then $\overline{V}^c = V_H$ and simple calculations show that

$$\overline{V}^c = \Pi^{\text{exp}} - \frac{1}{2}\delta\rho F.$$

[13] See the Appendix for a formal proof.

Thus, when the firms collude explicitly, they earn lower profits because of the fine imposed in the event the meeting is detected.

In order that the output q^c be produced along an equilibrium path it must be immune to all possible deviations. Note that any deviation is now systematically detected[14] and a firm may deviate in two ways: it may either raise its output or wave the meeting.

Consider first output deviations. A firm may cheat either in a period of no meeting or after the meeting has taken place. In both cases the most profitable deviation implies that a cheating firm optimally responds to the collusive output, q^c, and the no-deviation constraint is thus the same

$$(1-\delta)\underset{z}{Max}\ \pi^{\exp}(z,q^c) \leq (1-\delta)\Pi^{\exp}(q^c) + \frac{1}{2}\delta\left(V_H + V_L\right),$$

which is equivalent to

$$(1-\delta)\left[\underset{z}{Max}\ \pi^{\exp}(z,q^c) - \Pi^{\exp}(q^c)\right] \leq \delta\left(\Pi^{\exp}(q^c) - \frac{1}{2}\rho F\right). \qquad (\mathrm{IC}_q)$$

By comparing IC_{opn} and IC_q, one can see that the discounted value of the losses from abandoning collusion is lower when the firms collude explicitly. This is because when collusion breaks down, the firms stop communicating and hence are no longer exposed to the fine.

Now, if a firm deviates by waving the meeting it saves on the expected fine. The no-deviation constraint in this case takes the following form:

$$0 \leq (1-\delta)\left(\Pi^{\exp}(q^c) - \rho F\right) + \frac{1}{2}\delta\left(V_H + V_L\right),$$

which is equivalent to

$$(1-\delta)\left[\rho F - \Pi^{\exp}(q^c)\right] \leq \delta\left(\Pi^{\exp}(q^c) - \frac{1}{2}\rho F\right). \qquad (\mathrm{IC}_{meet})$$

As IC_q and IC_{meet} show, explicit collusion may not be sustained if the cost of communication, i.e., the expected fine, is too large. Therefore, I make

Assumption 3. $\rho F < 4\left(\frac{A}{12}\right)^2$.

This assumption states that the expected fine must not exceed the maximal joint collusive profit. As the Appendix shows, it ensures that there exists a range of discount factors where the firms can collude with communication. Furthermore, in such a case the IC_{meet} constraint is never binding and can thus be omitted.

[14] Cheating is either revealed by the current period price realization or during the next period meeting.

The analysis implies that in explicit collusion the firms choose q^c that solves the following program:

$$\begin{aligned} \overline{V}^c(\delta) = \underset{q^c}{Max} \Pi^{\exp}(q^c) - \tfrac{1}{2}\delta\rho F \\ s.t.\ \mathrm{IC}_q \text{ holds.} \end{aligned} \tag{P3}$$

The following proposition characterizes key properties of the solution to P3.

PROPOSITION 2. *There exist* δ_1' *and* δ_2' *such that* $0 < \delta_1' < \delta_2' < 1$ *and*
(i) *a solution to P3 exists only if* $\delta \in [\delta_1', 1)$,
(ii) IC_q *is binding in program P3 for any* $\delta \in [\delta_1', \delta_2')$.

Proof: see the Appendix.

The proposition thus states that explicit collusion cannot be sustained for small values of the discount factor. To gain the intuition of this result, recall that in this case even without communication a sustainable collusive output should be close to the static Nash equilibrium one. This implies that the benefits from collusion are small. Therefore if, in addition, communication is costly then explicit collusion may either not be profitable or not sustainable. On the other hand, when the discount factor is large then, as before, the firms can sustain the monopoly outcome. Hence, it is only for an intermediate range of discount factors that IC_q can be binding.

2.3 The optimal collusive scheme

This section derives the best collusive strategy. The analysis is based on the comparison of the value functions $\overline{V}^{nc}(\delta)$ and $\overline{V}^c(\delta)$ defined as the solutions, correspondingly, to programs P1 and P3.

PROPOSITION 3. *When* ρF *is not too large, then there exist an interval* $\Delta \subseteq (\widehat{\delta}_1, \widehat{\delta}_2)$ *such that* $\overline{V}^{nc}(\delta) < \overline{V}^c(\delta)$ *for any* $\delta \in \Delta$.

In words, when the expected fine is not too large then for some intermediate values of the discount factor the firms prefer collusion with communication to tacit collusion.

As this proposition is central to the paper, the proof is included in the text. To begin, consider programs P1 and P2. Since the objective functions in P1 and P2 coincide and the solution to P1 must satisfy an additional constraint, i.e., IC_{hdn}, then it must be $\overline{V}^{nc}(\delta) \leq \overline{V}(\delta)$. Note that one can have $\overline{V}^{nc}(\delta) < \overline{V}(\delta)$ only when the IC_{hdn} constraint is binding. Therefore, by applying proposition 1, one obtains

$$\overline{V}^{nc}(\delta) = \overline{V}(\delta) \text{ for any } \delta \in (0, \widehat{\delta}_1] \cup [\widehat{\delta}_2, 1), \tag{2}$$

$$\overline{V}^{nc}(\delta) < \overline{V}(\delta) \text{ for any } \delta \in (\widehat{\delta}_1, \widehat{\delta}_2). \tag{3}$$

Consider now programs P2 and P3. Since the objective function in P3 is lower than the one in P2 and the IC_q constraint is stronger than the IC_{opn} constraint then it must be $\overline{V}^c(\delta) < \overline{V}(\delta)$ for any $\rho F > 0$. Note also that IC_q is a continuous function of ρF and in the limit when ρF tends to zero IC_q and IC_{opn} coincide. Also, the objective function in P3 approaches $\Pi^{\exp}$ when $\rho F \to 0$. This implies that in the limit when $\rho F \to 0$ it must be that $\overline{V}^c(\delta)$ approaches $\overline{V}(\delta)$. It then follows that for sufficiently small values of ρF one can make the difference between $\overline{V}^c(\delta)$ and $\overline{V}(\delta)$ as small as desired. Given that $\overline{V}^{nc}(\delta)$ is lower than $\overline{V}(\delta)$ only for $\delta \in (\widehat{\delta}_1, \widehat{\delta}_2)$, there must exist $\delta' \in (\widehat{\delta}_1, \widehat{\delta}_2)$ such that $\overline{V}^{nc}(\delta') < \overline{V}^c(\delta')$ when ρF is sufficiently small. Since $\overline{V}^c(\delta)$ and $\overline{V}^{nc}(\delta)$ are continuous functions, there must also exist an interval Δ in the neighborhood of δ' such that $\overline{V}^{nc}(\delta) < \overline{V}^c(\delta)$ for any $\delta \in \Delta$. Finally, (3) implies $\Delta \subseteq (\widehat{\delta}_1, \widehat{\delta}_2)$. ■

In order to illustrate the result of proposition 3, simulations were performed. Figures 4 and 5 correspond to the case when $A = 10$, $\sigma = 0.4$, $\rho F = 0.3$ and $\rho F = 1$.

[Insert Figures 4 and 5 from the Appendix.]

As figure 4 shows, when the expected fine is sufficiently large, the average intertemporal payoff obtained in explicit collusion is always lower than the one obtained in tacit collusion. If instead the value of the expected fine is reduced, the firms can obtain a higher payoff in collusion with communication for some $\delta \in (\widehat{\delta}_1, \widehat{\delta}_2)$ as it is shown on figure 5. The simulations thus confirm the intuition. That is, in choosing the most profitable type of collusion the firms tradeoff information costs associated with imperfect observation of individual outputs against communication costs associated with the risk that the cartel is uncovered by the CA. Since information costs are absent for small and large values of the discount factor then it is only for some intermediate values of the discount factor the firms may prefer explicit collusion to tacit one.

3 Conclusion

The paper shows that communication can help sustain collusion. While there may be other reasons as for why collusive parties may want to meet, I have focused on the case when communication allows the firms to resolve uncertainty about past behavior. The analysis is based on the comparison of two collusive schemes: with and without communication. In tacit collusion the firms are hurt by the lack of complete information, whereas in explicit collusion they face the risk of being fined in the event the meeting is uncovered by the CA. The main finding of the paper is that as long as the punishment for illegal behavior is not too large, the optimal collusive scheme involves communication, when prices are low, for some range of discount factors.

Though the analysis has been performed for the case of a single additive stochastic shock, the results would be robust to alternative specifications of uncertainty. For example, one may think of a different probability distribution of the shock or multiple shocks which take on more than two values. What is crucial for the results obtained is that in all such cases one still maintains the assumption that communication eliminates uncertainty about past behavior which in turn implies the same tradeoff between informational costs and a legal fine.

The paper delivers some implications related to the cartel stability and the antitrust policy to fight collusion. Namely, it provides an economic rationale behind the meetings held by collusive firms and emphasizes the role of communication as a powerful mechanism to facilitate collusion. Used as a means to resolve uncertainty about individual actions taken in the past, it serves solely for the purpose of collusion. Finally, the paper suggests the explanation of why the firms may prefer explicit collusion and care less about the hard evidence left by the meeting. In the model under study, such collusive strategy appears to be optimal only if the expected punishment for cartel behavior is sufficiently small.

APPENDIX

A The Best Collusive Strategy in Case of Tacit Collusion

In this section the best collusive strategy is derived when firms choose tacit collusion. The proof is given for the case of symmetric (sequential) equilibria.

In general, a strategy Σ_i for firm i is a sequence of functions that specify an output production $q_{it} \in S$ at period t conditioning on firm i's own past outputs and past realizations of the random price $\widetilde{p} \in P(q_1 + q_2)$.

Let $U \in R_+$ denote the set of all symmetric (sequential) equilibrium payoffs V^{nc} of the game. U is nonempty because the strategy, Σ^{Nash}, that specifies playing the static Nash equilibrium outcome every period whatever the history is, constitutes a (sequential) equilibrium.

Define $\underline{V}^{nc} = \inf U$ and $\overline{V}^{nc} = \sup U$. Let $\overline{\Sigma}^{nc}$ and $\underline{\Sigma}^{nc}$ be the equilibrium strategies that correspond to $\overline{V}^{nc}$ and $\underline{V}^{nc}$, respectively.

Note that the present setting differs from the one developed by Abreu, et. al. (1986, 1990) in that the support of the price realization depends on firms' actions. To illustrate the main problem in this case, suppose that in the first period in a sequential equilibrium the firms are to produce q. If firm 1 cheats and some price $\widehat{p}$ outside the support $P(2q)$ is realized then firm 2 concludes that firm 1 has deviated but it cannot infer with probability one what the other firm's continuation strategy is: firm 1's continuation strategy may depend on its first-period action which is unobservable to firm 2.[15] The continuation profile need not be an equilibrium and a firm 1's continuation payoff can be even lower than the one obtained in the worst sequential equilibrium. This implies that the link between sequential equilibrium and admissibility of the continuation payoff with respect to U is broken after firms' own deviations.

Assumption 1, however, allows us to avoid this problem. Indeed, as is well known, any deviation is most effectively deterred when the worst punishment is inflicted, i.e., after any deviation a firm obtains its minmax payoff. Assumption 1 ensures that the minmax payoff is sustained by a reversal to the static Nash equilibrium, i.e., $\underline{\Sigma}^{nc} = \Sigma^{Nash}$ and $\underline{V}^{nc} = 0$, and thus implies that one can set the continuation payoff function equal to zero whenever the realized price $\widetilde{p}$ falls outside the set of equilibrium prices $P(2q)$.

Consider now a symmetric equilibrium strategy where each firm makes its actions depend only upon past signal realizations. As it is shown in Abreu, et. al. (1986, 1990), any symmetric strategy equilibrium profile can then be factored into a single-period symmetric output q and a continuation payoff function $V : R_+ \to R_+$, such that $V(\widetilde{p}) \in U$ for any $\widetilde{p} \in P$ and

$$\begin{aligned} & (1-\delta)\Pi^{\exp}(q) + \delta E\left[V(\widetilde{p}(2q)\right] \\ \geq \;& (1-\delta)\pi^{\exp}(z,q) + \delta E\left[V(\widetilde{p}(z+q))\right] \text{ for } z \in S. \end{aligned} \tag{A1}$$

Using the fact that factorization (A1) holds, I can prove the following lemma.

Lemma 1. U is compact.

Proof: Let W be an arbitrary set. We say that a pair $(q, V(\cdot))$ is admissible with respect to W if

(i) $V(p) \in co(W)$[16] for any $p \in [0, p_{\max}]$,

(ii) (A1) is satisfied.

Define the operator $B(W)$ as follows:

$$\begin{aligned} B(W) &= \{w \in R : w = (1-\delta)\Pi^{\exp}(q) + \delta E\left[V(\widetilde{p}(2q)\right], \\ & \quad (q, V(\cdot)) \text{ is admissible w. r. t. } W\}. \end{aligned}$$

According to Abreu, et. al. (1986, 1990), U is the largest bounded invariant set generated by the operator B, i.e., $U = B(U)$. Hence, in order to prove compactness of U it suffices to show that B is compact.

[15] When the shock takes on two values, $\widehat{p}$ can result either from an output expansion and the low demand state or from an output contraction and the high demand state.

[16] We assume that there exists a public randomization device that convexifies the set W.

Let $W \subset R$ be a nonempty and compact set. Since $\Pi^{\exp}(\cdot)$ is bounded then $B(W)$ is also bounded.

To show that $B(W)$ is closed, consider a sequence $\{w^k\} \subset B(W)$ which converges to some w^∞. I need to show that $w^\infty \in B(W)$.

By the definition of $B(W)$, there exists a pair $(q^k, V^k(\cdot))$ which is admissible w.r.t. W and obtains the value w^k. Recall that $\widetilde{p}(2q) \in \{\, p_H(2q), p_L(2q)\}$. Denote $V_H^k = V^k(p_H(2q^k))$, $V_L^k = V^k(p_L(2q^k))$ and $w_{\min} = \min W$ and define $\widehat{V}^k(\cdot)$ as follows:

$$\widehat{V}^k(p) = \begin{cases} V_l^k, & \text{if } p = p_l(2q^k),\ l = H, L \\ w_{\min}, & \text{otherwise.} \end{cases}$$

Thus, $(q^k, \widehat{V}^k(\cdot))$ is equivalent to the vector $(q^k, V_H^k, V_L^k, w_{\min})$. It is straightforward to see that the pair $(q^k, \widehat{V}^k(\cdot))$ is admissible w.r.t. W and delivers the same value w^k, i.e.,

$$w^k = (1-\delta)\Pi^{\exp}(q^k) + \frac{1}{2}\delta(V_H^k + V_L^k).$$

Let us denote

$$G(q^k, V_H^k, V_L^k) \equiv \max_{z \in S}(1-\delta)\pi^{\exp}(z, q^k) + \delta E\left[\widehat{V}^k(\widetilde{p}(z + q^k))\right].$$

Denote r_H an output such that when the demand is high, the realized market price corresponds to the low demand state, i.e.,

$$r_H(q)\ :\ p_H(r_H + q) = p_L(2q).$$

In the same way, denote r_L an output such that when the demand is low, the realized market price corresponds to the high demand state, i.e.,

$$r_L(q)\ :\ p_L(r_L + q) = p_H(2q).$$

It is easy to verify that $r_H(q) = 2\sigma + q$ and $r_L(q) = q - 2\sigma$.Define

$$z^*(q) = \arg\max_{s \in S} \pi^{\exp}(z, q) = \max\{\frac{1}{2}(A - q), 0).$$

I can write now

$$\begin{aligned} G(q^k, V_H^k, V_L^k) \quad = \quad & \max\Big\{(1-\delta)\pi^{\exp}(z^*(q^k), q^k) + \delta w_{\min}, \\ & (1-\delta)\pi^{\exp}(r_H(q^k), q^k) + \frac{1}{2}\delta(V_L^k + w_{\min}), \\ & (1-\delta)\pi^{\exp}(r_L(q^k), q^k) + \frac{1}{2}\delta(V_H^k + w_{\min})\Big\}. \end{aligned} \tag{A2}$$

Since $\pi^{\exp}(\cdot)$ is continuos then, as (A2) implies, $G(q^k, V_H^k, V_L^k)$ is a continuos function of $\left(q^k, V_H^k, V_L^k\right)$. Similarly, since $\Pi^{\exp}(\cdot)$ is continuos then w^k is a continuous function of $\left(q^k, V_H^k, V_L^k\right)$.

The analysis so far implies that if a sequence $\left(q^k, V_H^k, V_L^k\right)$ converges to $(q^\infty, V_H^\infty, V_L^\infty)$ then $w^k \to w^\infty$. The fact that $\left(q^k, V_H^k, V_L^k\right)$ is admissible w.r.t. W implies

$$(1-\delta)\Pi^{\exp}(q^k) + \frac{1}{2}\delta(V_H^k + V_L^k) \geq G(q^k, V_H^k, V_L^k).$$

Since both sides of the above inequality are continuos functions of $\left(q^k, V_H^k, V_L^k\right)$ then the limit, $(q^\infty, V_H^\infty, V_L^\infty)$ is admissible w.r.t. W. That proves $w^\infty \in B(W)$ and, therefore, $B(W)$ is compact. ■

Since U is compact then $\overline{V}^{nc} \in U$. Consider the following strategy $\Sigma^{nc}(q)$.[17]

(i) At $t = 0$ produce q.

(ii) For any $t \geq 1$: play $\overline{\Sigma}^{nc}$ if in the previous period the realized price $\widetilde{p} \in P(2q)$, otherwise play Σ^{Nash}.

Lemma 2. If the collusive output is equal to q at some point on an equilibrium path then $\Sigma^{nc}(q)$ is an equilibrium strategy.

Proof: Let $V^{nc}(q)$ denote the payoff obtained from the strategy $\Sigma^{nc}(q)$, that is

$$V^{nc}(q) \equiv (1-\delta)\Pi^{\exp}(q) + \delta\overline{V}^{nc}. \tag{A3}$$

In order that $\Sigma^{nc}(q)$ be an equilibrium strategy, it must be immune to all possible deviations.

Open deviations

Denote $\widetilde{p}(z+q) \equiv a + \widetilde{\theta} - (z+q)$, i.e., $\widetilde{p}(z+q)$ is the realized price when a deviating firm produces z.

In the open deviation a firm chooses z such that $\widetilde{p}(z+q) \notin P(2q)$ and earns $\pi^{\exp}(z,q)$. Using Σ^{Nash} as the punishment, the no-open deviation constraint thus takes the form:

$$(1-\delta)\left[\pi^{\exp}(z,q) - \Pi^{\exp}(q)\right] \leq \delta\overline{V}^{nc} \text{ for any } z \text{ s.t. } \widetilde{p}(z+q) \notin P(2q). \tag{A4}$$

Now, I show that if q satisfies (A1) then (A4) must hold. Indeed, (A1) implies

[17] In general, one could consider strategies when, along the equilibrium paths, the actions are contingent on the realized shocks (e.g. for $\widetilde{p} = p_H(2q)$ and $\widetilde{p} = p_L(2q)$ firms chose different quantities). The uniform distribution of the shocks allows us to restrict attention on simpler strategies of the form $\Sigma^{nc}(q)$.

$$(1-\delta)\left[\pi^{\exp}(z,q)-\Pi^{\exp}(q)\right]\leq\delta E\left[V(\widetilde{p}(2q))-V(\widetilde{p}(z+q))\right]\text{ for any }z.$$

Using the definition of $\overline{V}^{nc}$ and the fact that $\underline{V}^{nc}=0$, one obtains

$$V(\widetilde{p}(2q))-V(\widetilde{p}(z+q))\leq\overline{V}^{nc}\text{ for any }z.$$

This establishes (A4).

Hidden deviations

Two cases must be considered. First, a cheating firm may deviate by suboptimally expanding its output production in order to mimic the price of the low-demand state when in fact the demand is high. In which case it produces $r_H(q)$ which solves the following equation:

$$p_H(r_H(q)+q)=p_L(2q).$$

It can be easily verified that $r_H(q)=2\sigma+q$ and the one-period profit from cheating is

$$\pi^{\exp}(r_H(q),q)=\Pi^{\exp}(q)+6\sigma\left[\left(q^{Nash}-q\right)-\frac{2}{3}\sigma\right]. \tag{A5}$$

The intertemporal payoff from such deviation is

$$(1-\delta)\pi^{\exp}(r_H(q),q)+\delta\left(\frac{1}{2}\overline{V}^{nc}+\frac{1}{2}\times 0\right).$$

The no-deviation constraint thus takes the form

$$(1-\delta)\left[\pi^{\exp}(r_H(q),q)-\Pi^{\exp}(q)\right]\leq\delta\frac{1}{2}\overline{V}^{nc}. \tag{A6}$$

Now, I show that if q satisfies (A1) then (A6) must hold. Indeed, (A1), in particular, implies

$$(1-\delta)\left[\pi^{\exp}(r_H(q),q)-\Pi^{\exp}(q)\right]\leq\delta E\left[V(\widetilde{p}(2q))-V(\widetilde{p}(r_H(q)+q))\right].$$

Given that the shock is uniformly distributed and $V(p_H(r_H(q)+q))=V(p_L(2q))$, one obtains

$$E\left[V(\widetilde{p}(2q))-V(\widetilde{p}(r_H(q)+q))\right]=\frac{1}{2}V(p_H(2q))-\frac{1}{2}V(p_L(r_H(q)+q))\leq\frac{1}{2}\overline{V}^{nc}.$$

Thus, (A6) is established.

If a cheating firm suboptimally contracts its output in order to mimic the high-demand when in fact the demand is low then it produces $r_L(q)$ which solves the following equation:

$$p_L(q - r_L(q)) = p_H(2q).$$

One easily verifies that $r_L(q) = q - 2\sigma$ and the one-period profit from cheating is

$$\pi^{\exp}(r_L(q), q) = \Pi^{\exp}(q) - 6\sigma\left[\left(q^{Nash} - q\right) + \frac{2}{3}\sigma\right].$$

Since $\pi^{\exp}(r_L(q), q) < \Pi^{\exp}(q)$, such deviations induce losses and therefore are never profitable. ■

The following lemma says that, without loss of generality, in searching for the best collusive strategy one can restrict attention on studying $\Sigma^{nc}(q)$ only.

Lemma 3. The set of outputs q that can be sustained at some point on an equilibrium path is a closed interval I, which is the set of outputs satisfying

$$(1-\delta)\left[\underset{z}{Max}\,\pi^{\exp}(z, q) - \Pi^{\exp}(q)\right] \leq \delta\overline{V}^{nc}, \tag{A7}$$

$$(1-\delta)\left[\pi^{\exp}(r_H(q), q) - \Pi^{\exp}(q)\right] \leq \delta\frac{1}{2}\overline{V}^{cn}. \tag{A8}$$

Proof: From lemma 2, it follows that if q is sustained at some point on an equilibrium path, then

$$(1-\delta)Max\left\{\sup_{\widetilde{p}(z+q)\notin P(2q)}\left[\pi^{\exp}(z,q) - \Pi^{\exp}(q)\right], 2\left[\pi^{\exp}(r_H(q), q) - \Pi^{\exp}(q)\right]\right\} \leq \delta\overline{V}^{nc}. \tag{A9}$$

Conversely, if this condition is satisfied for some q then $\Sigma^{nc}(q)$ is an equilibrium strategy in which firms produce q in the first period. Therefore, an output q can be sustained at some point in an equilibrium if and only if it satisfies (A9).

Let $z^*(q) \equiv \arg\max_z \pi^{\exp}(z, q)$. Following lemma 2, that it can never be $z^*(q) = r_L(q)$. Now, if $z^*(q) \neq r_H(q)$ then (A9) and the pair (A7)-(A8) are clearly equivalent. If $z^*(q) = r_H(q)$ then (A8) becomes stronger than (A7) and coincides with (A9). That proves that overall (A9) and the pair (A7)-(A8) are equivalent.

Lastly, since the left-hand sides of (A7) and (A8) are convex functions of q, the set of quantities satisfying both constraints is a closed interval. ■

Define

$$q^* = \arg\max_{q\in I}\Pi^{\exp}(q). \tag{A10}$$

The following lemma states that in searching for the most profitable collusive strategy, one can restrict attention to the class of stationary equilibria.

Lemma 4. $\overline{\Sigma}^{nc}$ *is a stationary equilibrium strategy.*

Proof: Since $\Sigma^{nc}(q^*)$ is some equilibrium strategy and $\overline{V}^{nc}$ is the highest equilibrium payoff, then, by using (A3), one obtains

$$V^{nc}(q^*) = (1-\delta)\Pi^{\exp}(q^*) + \delta\overline{V}^{nc} \leq \overline{V}^{nc},$$

which implies $\Pi^{\exp}(q^*) \leq \overline{V}^{nc}$.

Now, let $\{q_t^{nc}\}_{t=0}^{\infty}$ denote the profile of outputs induced by the strategy $\overline{\Sigma}^{nc}$. Since $\overline{\Sigma}^{nc}$ is an equilibrium strategy then, according to Lemma 3, q_t^{nc} must satisfy (A7)-(A8) for any t. Then, from the maximization task (A10) it follows that $\Pi^{\exp}(q^*) \geq \Pi^{\exp}(q_t^{nc})$ for any t and, therefore, $\overline{V}^{nc} \leq \Pi^{\exp}(q^*)$. I thus have

$$\overline{V}^{nc} \leq \Pi^{\exp}(q^*) \leq \overline{V}^{nc},$$

implying $\Pi^{\exp}(q^*) = \overline{V}^{nc}$, which is possible only if the firms produce q^* at every period. Thus, the best collusive equilibrium strategy $\overline{\Sigma}^{nc}$ defines a stationary path along which each firm produces q^* at every point of time. ■

Lemma 5. The maximum equilibrium payoff $\overline{V}^{nc}$ *is the solution to*

$$\begin{aligned}\overline{V}^{nc} &= \underset{q}{Max}\,\Pi^{\exp}(q) \\ s.t.(1-\delta)\left[\underset{z}{Max}\ \pi^{\exp}(z,q) - \Pi^{\exp}(q)\right] &\leq \delta\Pi^{\exp}(q), \\ (1-\delta)\left[\pi^{\exp}(r_H(q^{nc}),q^{nc}) - \Pi^{\exp}(q^{nc})\right] &\leq \delta\tfrac{1}{2}\Pi^{\exp}(q^{nc}).\end{aligned} \tag{A11}$$

Proof: Since $\overline{\Sigma}^{nc}$ is stationary, then the constraints in (A11) are the no-deviation constraints to be satisfied along the equilibrium path. Let q^{nc} be the solution to program (A11). One needs to prove that $q^{nc} = q^*$.

On the one hand, since $\overline{V}^{nc}$ is the highest equilibrium payoff, then $\Pi^{\exp}(q^*) \leq \overline{V}^{nc} = \Pi^{\exp}(q^{nc})$. On the other hand, since $q^{nc} \in I$, then from (A10) it follows that $\Pi^{\exp}(q^{nc}) \leq \Pi^{\exp}(q^*)$. Thus, $\Pi^{\exp}(q^{nc}) = \Pi^{\exp}(q^*)$ and the proof is complete. ■

B The Proof of Proposition 1

Given the linear demand function (1), the profit from the open deviation is

$$\underset{z}{Max}\,\pi^{\exp}(z,q^{nc}) = \Pi^{\exp}(q^{nc}) + \frac{9}{4}\left[q^{Nash} - q^{nc}\right]^2. \tag{B1}$$

Recall that $\Pi^{\exp}(q^{nc})$ is the difference between the profit and the static Nash profit and thus can be written as

$$\Pi^{\exp}(q^{nc}) = 2\left[q^{Nash} - q^{nc}\right]\left[\frac{1}{6}A - \left(q^{Nash} - q^{nc}\right)\right]. \tag{B2}$$

Let us denote $s = q^{Nash} - q^{nc}$. By using (A4), (B1) and (B2), I rewrite the IC_{opn} and IC_{hdn} constraints in the following form

$$\frac{9}{4}s^2 \leq \frac{\delta}{1-\delta}2s\left(\frac{1}{6}A - s\right), \tag{$\widetilde{\mathrm{IC}}_{opn}$}$$

$$12\sigma\left(s - \frac{2}{3}\sigma\right) \leq \frac{\delta}{1-\delta}2s\left(\frac{1}{6}A - s\right). \tag{$\widetilde{\mathrm{IC}}_{hdn}$}$$

Notice that for any $s < 0$ the $\widetilde{\mathrm{IC}}_{opn}$ constraint can never be satisfied, thus I consider only $s \geq 0$ (which implies $q^{nc} \leq q^{Nash}$). Now, program P1 can be stated as

$$\begin{array}{c} \underset{s\geq 0}{Max} \frac{\delta}{1-\delta}2s\left(\frac{1}{6}A - s\right) \\ \text{s.t. } \widetilde{\mathrm{IC}}_{opn} \text{ and } \widetilde{\mathrm{IC}}_{hdn} \text{ hold.} \end{array} \tag{B3}$$

Define $S(\sigma)$ the set of s such that $s \geq 0$ and the $\widetilde{\mathrm{IC}}_{hdn}$ constraint is stronger than the $\widetilde{\mathrm{IC}}_{opn}$ one. By comparing the left hand sides of $\widetilde{\mathrm{IC}}_{hdn}$ and $\widetilde{\mathrm{IC}}_{opn}$ I find that $S(\sigma) = [\underline{s}, \overline{s}]$, where

$$\underline{s}(\sigma) = \frac{8}{3}\left(1 - \frac{1}{\sqrt{2}}\right)\sigma \quad \text{and} \quad \overline{s}(\sigma) = \frac{8}{3}\left(1 + \frac{1}{\sqrt{2}}\right)\sigma.$$

For a given σ, the graphs of $\frac{9}{4}s^2$ and $12\sigma\left(s - \frac{2}{3}\sigma\right)$ are drawn on figure 6.

Denote $u(s,\delta) \equiv \frac{\delta}{1-\delta}2s\left(\frac{1}{6}A - s\right)$, then the solution to unconstrained maximization of $u(s,\delta)$ is $\frac{1}{12}A$, which corresponds to the monopoly outcome, $q^{nc} = \frac{1}{4}A = \frac{1}{2}q^M$.

Figure 6 illustrates different positions of the graph of $u(s,\delta)$ for different values of δ and given that A is fixed (here A is chosen such that $\underline{s} < \frac{1}{12}A < \overline{s}$). Notice that the graph of $u(s,\delta)$ shifts upward when δ rises, i.e., for any δ_1, δ_2 and δ_3 such that $\delta_1 < \delta_2 < \delta_3$ one obtains $u_1 < u_2 < u_3$ where $u_i = u(s,\delta_i)$.

The solution, s^{nc}, to (B3) is obtained as follows. Fix a value of A, then s^{nc} is either equal to $\frac{1}{12}A$ or obtained as the point of intersection between the curve of $u(s,\delta)$ and the upper frontier composed of the curves of $\frac{9}{4}s^2$ and $12\sigma\left(s - \frac{2}{3}\sigma\right)$. Figure 6 thus shows which constraint, if any, in program B3 is binding for given δ.

Let us define $s_{opn}(\delta)$ and $s_{hdn}(\delta)$ as the solutions to the two no-deviation constraints when they are binding

$$s_{opn}(\delta) : \frac{9}{4}s^2 = \frac{\delta}{1-\delta}2s\left(\frac{1}{6}A - s\right),$$

$$s_{hdn}(\delta) : 12\sigma\left(s - \frac{2}{3}\sigma\right) = \frac{\delta}{1-\delta}2s\left(\frac{1}{6}A - s\right).$$

It can be verified that $s_{opn}(\delta)$ and $s_{hdn}(\delta)$ are continuously increasing functions of δ.

Depending on the value of A, three cases are possible.

Case 1. $\underline{s} < \frac{1}{12}A < \overline{s}$ or equivalently

$$\frac{1}{32\left(1 + \frac{1}{\sqrt{2}}\right)}A < \sigma < \frac{1}{32\left(1 - \frac{1}{\sqrt{2}}\right)}A.$$

In this case $u(s,\delta)$ attains its maximum at the point which belongs to the interval $(\underline{s}, \overline{s})$, as depicted on figure 6. When δ is small enough the graph of $u(s,\delta)$ is the u_1-curve which, as figure 6 shows, first crosses the $\widetilde{\mathrm{IC}}_{opn}$ constraint. This implies that the solution is $s^{nc} = s_{opn}$. As long as δ increases, the graph of $u(s,\delta)$ shifts upward. For some values of δ, it is as depicted by the u_2-curve and first crosses the $\widetilde{\mathrm{IC}}_{hdn}$ constraint. This implies $s^{nc} = s_{hdn}$. When δ rises further, the graph of $u(s,\delta)$ is the u_3-curve, which in turn implies that both constraints in (B3) are slacked. The solution is then $s^{nc} = \frac{1}{12}A$.

Define $\widehat{\delta}_1$ and $\widehat{\delta}_2$ as solutions to the following equations:

$$\widehat{\delta}_1 : s_{opn}(\widehat{\delta}_1) = s_{hdn}(\widehat{\delta}_1) = \underline{s},$$

$$\widehat{\delta}_2 : s_{hdn}(\widehat{\delta}_2) = \frac{1}{12}A.$$

Since $s_{hdn}(\delta)$ is continuous then such $\widehat{\delta}_1$ and $\widehat{\delta}_2$ exist. Using the fact that $s_{hdn}(\delta)$ is an increasing function of δ and $\underline{s} < \frac{1}{12}A$, I obtain $\widehat{\delta}_1 < \widehat{\delta}_2$.

The analysis thus implies that in program B3

- the no-open deviation constraint is binding for any $\delta \in \left(0, \widehat{\delta}_1\right]$,
- the no-hidden deviation constraint is binding for any $\delta \in \left(\widehat{\delta}_1, \widehat{\delta}_2\right)$,
- both constraints are relaxed for any $\delta \in \left[\widehat{\delta}_2, 1\right)$, and the solution to (B3) is

$$s^{nc}(\delta) = \min\{s_{opn}(\delta), s_{hdn}(\delta), \frac{1}{12}A\}. \tag{B4}$$

Case 2. $\overline{s} \leq \frac{1}{12}A$ or equivalently

$$\sigma \leq \frac{1}{32\left(1+\frac{1}{\sqrt{2}}\right)} A.$$

By applying the same reasoning as in Case 1, I obtain $s^{nc} = s_{opn}$ for small δ's, $s^{nc} = s_{hdn}$ for some moderate δ's, $s^{nc} = s_{opn}$ for large δ's and $s^{nc} = \frac{1}{12}A$ when δ is very large.

Define $\widehat{\delta}_1$ and $\widehat{\delta}_2$ as follows[18]

$$\widehat{\delta}_1 : s_{opn}(\widehat{\delta}_1) = s_{hdn}(\widehat{\delta}_1) = \underline{s},$$

$$\widehat{\delta}_2 : s_{opn}(\widehat{\delta}_2) = s_{hdn}(\widehat{\delta}_2) = \overline{s},$$

$$s_{opn}(\delta = \frac{9}{17}) = \frac{1}{12}A.$$

Since $s_{hdn}(\delta)$ and $s_{opn}(\delta)$ are continuous then such $\widehat{\delta}_1$ and $\widehat{\delta}_2$ exist. Using the fact that $s_{hdn}(\delta)$ and $s_{opn}(\delta)$ are increasing functions of δ and $\underline{s} < \overline{s} < \frac{1}{12}A$, I obtain $\widehat{\delta}_1 < \widehat{\delta}_2 < \frac{9}{17}$.

The analysis thus implies that in program B3

- the no-open deviation constraint is binding for any $\delta \in \left(0, \widehat{\delta}_1\right] \cup \left[\widehat{\delta}_2, \frac{9}{17}\right)$,
- the no-hidden deviation constraint is binding for any $\delta \in \left(\widehat{\delta}_1, \widehat{\delta}_2\right)$,
- both constraints are relaxed for any $\delta \in \left[\frac{9}{17}, 1\right)$ and the solution to (B3) is given by (B4).

Case 3. $\frac{1}{12}A \leq \underline{s}$ or equivalently

$$\sigma \geq \frac{1}{32\left(1-\frac{1}{\sqrt{2}}\right)} A.$$

As figure 6 shows, in this case $\widetilde{\mathrm{IC}}_{hdn}$ is never be binding and therefore uncertainty does not impede collusion. The solution is

$$s^{nc}(\delta) = \min\{s_{opn}(\delta), \frac{1}{12}A\}.$$

■

C The Proof of Proposition 2

[18] Notice that $\widehat{\delta}_1$ is different in Case 2 and Case 3 because $\underline{s}$ depends on σ.

Denote $s = q^{Nash} - q^c$ and $u_F(s,\delta) \equiv \frac{\delta}{1-\delta}\left[2s\left(\frac{1}{6}A - s\right) - \frac{1}{2}\rho F\right]$. By using (B1) and (B2), I rewrite the IC_q and IC_{meet} constraints as follows

$$\frac{9}{4}s^2 \leq u_F(s,\delta), \tag{$\widetilde{\mathrm{IC}}_q$}$$

$$\rho F - 2s\left(\frac{1}{6}A - s\right) \leq u_F(s,\delta). \tag{$\widetilde{\mathrm{IC}}_{meet}$}$$

Notice that if $s \leq 0$ then $u_F(s,\delta) < 0$ and thus the $\widetilde{\mathrm{IC}}_q$ constraint can never be satisfied. Therefore, I consider only $s > 0$ (which implies $q^c \leq q^{Nash}$). Now, program P3 can be stated as

$$\begin{gathered} \underset{s}{Max} u_F(s,\delta) \\ s.t.\ \widetilde{\mathrm{IC}}_q \text{ and } \widetilde{\mathrm{IC}}_{meet} \text{ hold.} \end{gathered} \tag{C1}$$

First of all, I show that the $\widetilde{\mathrm{IC}}_{meet}$ constraint in C1 can be omitted. Define $S_F = \{s : s \geq 0 \text{ and } \rho F - 2s\left(\frac{1}{6}A - s\right) \geq \frac{9}{4}s^2\}$. One can verify that $S_F = [0, s_F]$, where

$$s_F = \frac{2}{3}A\left[\sqrt{1 + 9\frac{\rho F}{A^2}} - 1\right].$$

Assumption 3 ensures that $s_F < \frac{1}{12}A$. Suppose now that $\widetilde{\mathrm{IC}}_{meet}$ in C1 is binding for some s' then it must be $s' < s_F$. Since $u_F(s,\delta)$ is concave and attains its maximum at the point $\frac{1}{12}A$ while $\rho F - 2s\left(\frac{1}{6}A - s\right)$ is decreasing on the interval $\left(0, \frac{1}{12}A\right)$ then a slight increase of s' only weakens $\widetilde{\mathrm{IC}}_{meet}$ and at the same time raises the value of $u_F(s,\delta)$ in C1. Hence, it is never optimal to have $\widetilde{\mathrm{IC}}_{meet}$ binding in C1.

To illustrate the solution to program C1, I refer to figure 7. Before proceeding, note that assumption 3 ensures that there exists a non empty set of s where $u_F(s,\delta) > 0$.

Figure 7 shows the graph of the $\widetilde{\mathrm{IC}}_q$ constraint, i.e., the curve of $\frac{9}{4}s^2$, and different positions of the $u_F(s,\delta)$-curve depending on the value of δ. Notice also that $u_F(s,\delta)$ is increasing with δ.

If δ is small enough then $u_F(s,\delta)$ is located below $\frac{9}{4}s^2$, as depicted by the u_{F1}-curve. This implies that there is no $s \geq 0$ such that $\widetilde{\mathrm{IC}}_q$ is satisfied and collusion is thus impossible.

When δ increases, $u_F(s,\delta)$ shifts upward. As figure 7 makes it clear, there exists δ_1' such that for any $\delta > \delta_1'$ the $\widetilde{\mathrm{IC}}_q$ constraint can be satisfied. δ_1' is obtained when $u_F(s,\delta_1')$ is tangent to $\frac{9}{4}s^2$, as depicted by the u_{F2}-curve. Direct calculations lead to

$$\delta_1' = \frac{9}{1 + \frac{A^2}{4\rho F}}.$$

When δ rises slightly above δ_1', the $\widetilde{\mathrm{IC}}_q$ constraint in C1 becomes binding. The graph of $u_F(s,\delta)$ is the u_{F3}-curve and the solution, $s^n = s_q(\delta)$, is defined as the largest root of the following equation:

$$\frac{9}{4}s^2 = \frac{\delta}{1-\delta}\left[2s\left(\frac{1}{6}A - s\right) - \frac{1}{2}\rho F\right].$$

A further increase of δ above δ_2' leads to the situation where the graph of $u_F(s,\delta)$ is the u_{F4}-curve, i.e., the $\widetilde{\mathrm{IC}}_q$ constraint is relaxed. In this case, $s^c = \frac{1}{12}A$ and firms can thus sustain the monopoly outcome.

Finally, δ_2' is obtained from the following equation:

$$\delta_2' : s_q(\delta) = \frac{1}{12}A.$$

Direct calculations lead to

$$\delta_2' = \frac{9}{17 - 288\frac{\rho F}{A^2}}$$

One can verify that $\delta_1' < \delta_2' < 1$, and the proof is complete. ■

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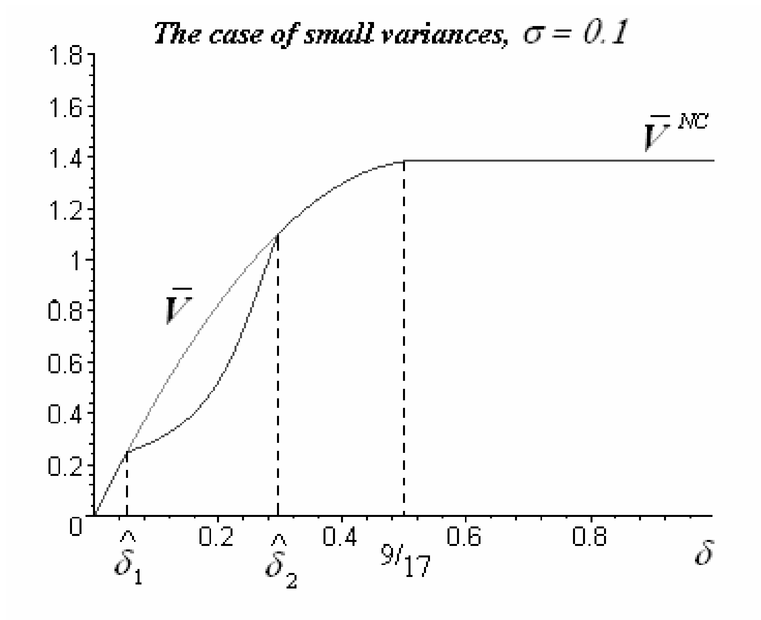
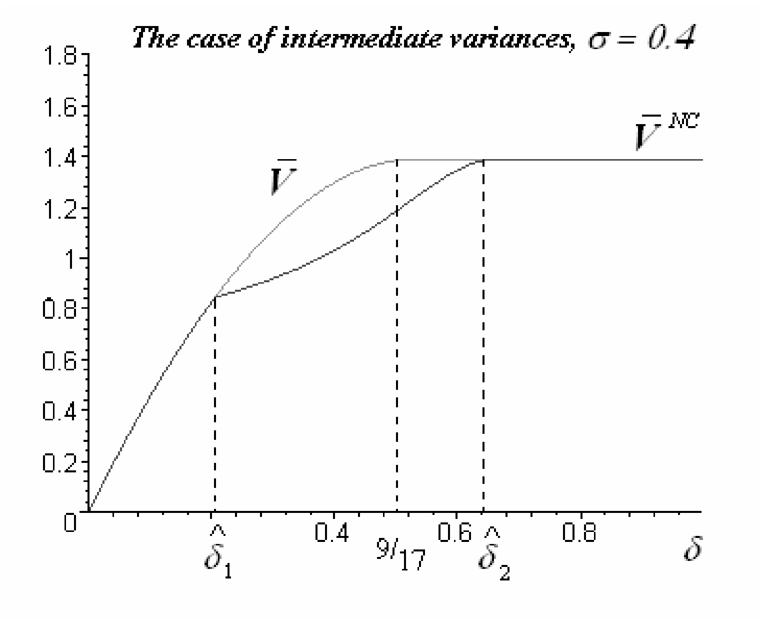


Figure 1:



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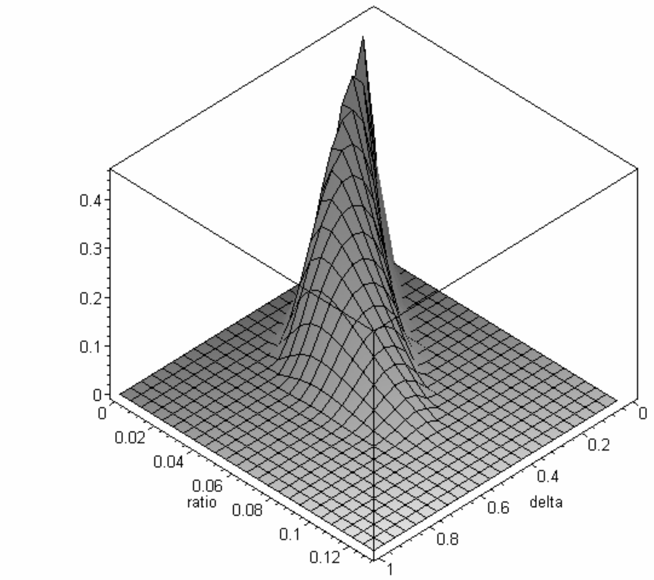

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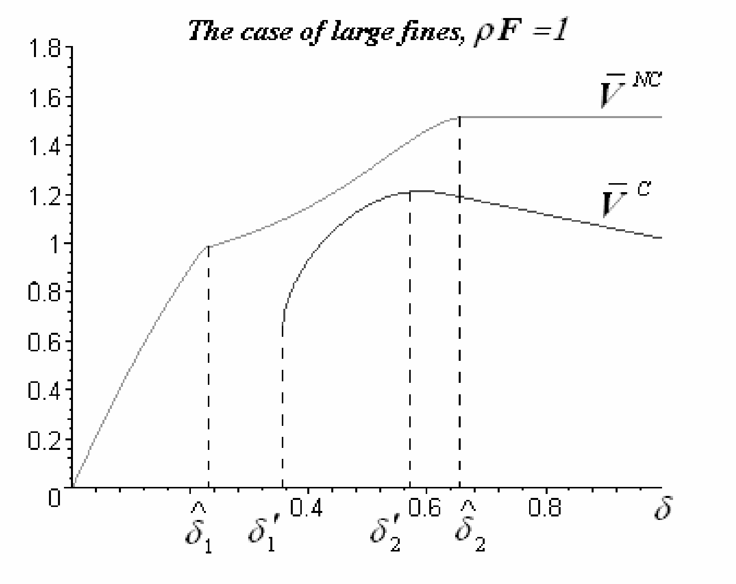

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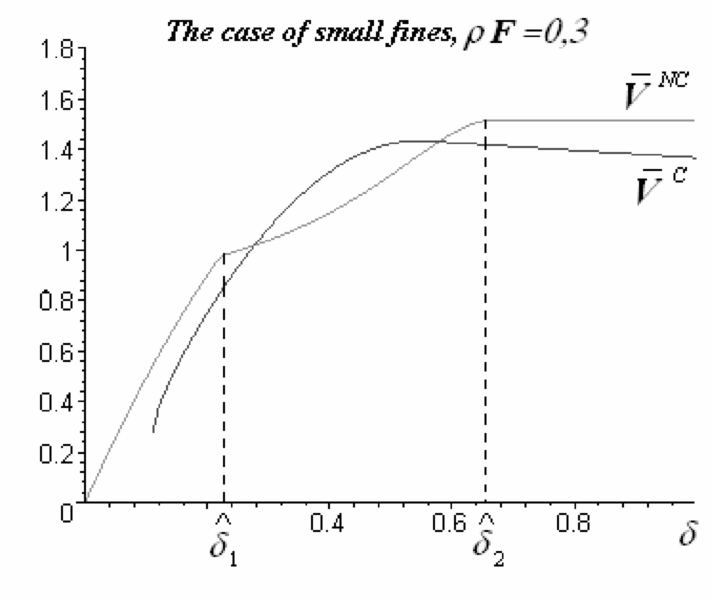

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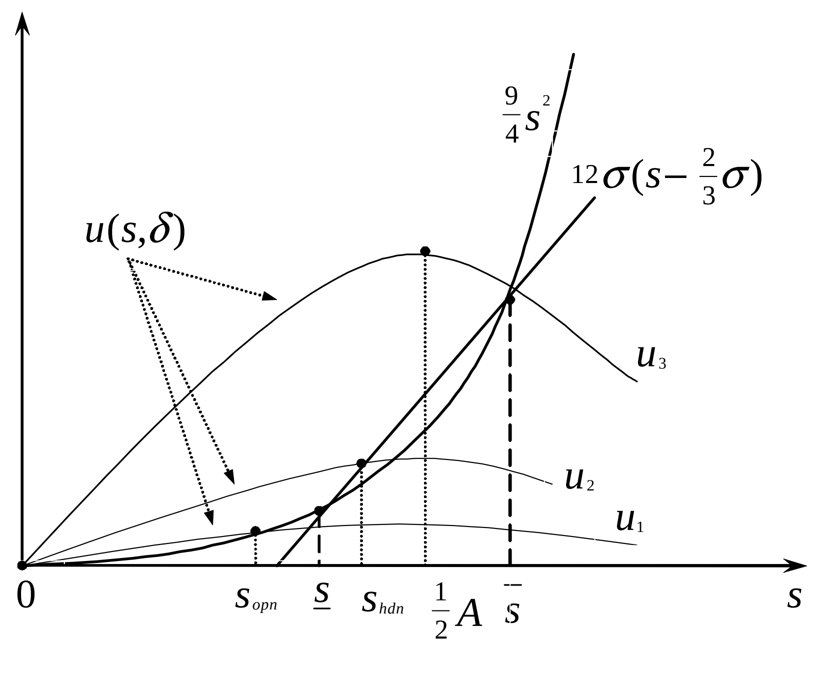

Figure 6:

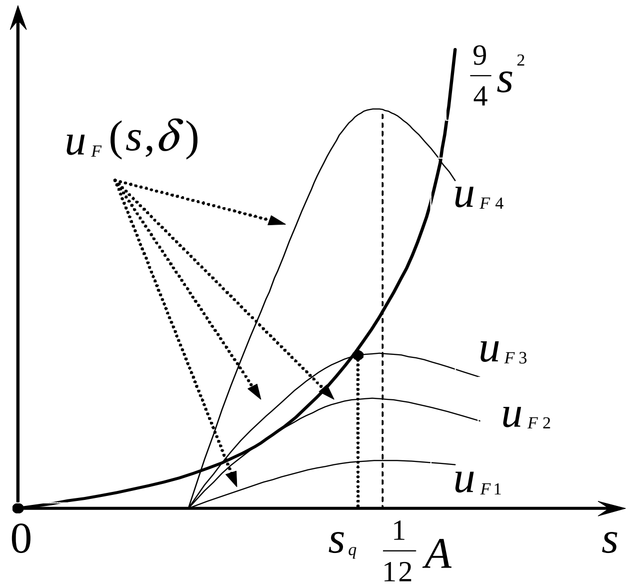

Figure 7: